Chapter 3
ESTIMATION OF NEARSHORE WAVES

Table of Contents

List of Tables

List of Figures

Chapter II-3
Estimation of Nearshore Waves

II-3-1. Introduction

a. Background.

(1) Coastal engineering considers problems near the shoreline normally in water depths of less than 20 m. Project designs usually require knowledge of the wave field over an area of 1-10 km^2 in which the depth may vary significantly. Additionally, study of shoreline change and beach protection frequently requires analysis of coastal processes over entire littoral cells, which may span 10-100 km in length. Wave data are generally not available at the site or depths required. Often a coastal engineer will find that data have been collected or hindcast at sites offshore in deeper water or nearby in similar water depths. This chapter provides procedures for transforming waves from offshore or nearby locations to nearshore locations needed by the engineer.

(2) Understanding the processes that affect coastal waves is essential to coastal engineering. Waves propagating through shallow water are strongly influenced by the underlying bathymetry and currents (Figure II-3-1). A sloping or undulating bottom, or a bottom characterized by shoals or underwater canyons, can cause large changes in wave height and direction of travel. Shoals can focus waves, in some cases more than doubling wave height behind the shoal. Other bathymetric features can reduce wave heights. The magnitude of these changes is particularly sensitive to wave period and direction and how the wave energy is spread in frequency and direction (Figure II-3-2). In addition, wave interaction with the bottom can cause wave attenuation. The influence of bathymetry on local wave conditions cannot be overstated as a critical factor in coastal engineering design.

(3) Wave height is often the most significant factor influencing a project. Designing with a wave height that is overly conservative can greatly increase the cost of a project and may make it uneconomical. Conversely, underestimating wave height could result in catastrophic failure of a project or significant maintenance costs. Approaches for transforming waves are numerous and differ in complexity and accuracy. Consequently transformation studies require careful analysis. They are but one part of selecting project design criteria, which will be treated in Part II-9.

(4) Wave transformation across irregular bathymetry is complex. Simplifying assumptions admit valid and useful approximations for estimating nearshore waves. After this introduction, a basic principles section provides an overview of the theoretical basis for wave transformation analyses, followed by development of a simple method for refraction and shoaling estimates. Transformation of irregular waves is then discussed. Next, advanced wave transformation models currently used by the Corps of Engineers are discussed. A final section provides guidance on selecting the approach used in calculating wave transformation. This chapter is primarily directed at open coast wave problems excluding structures such as breakwaters or jetties. Analyses involving structures are provided in Part II-7.

b. Practical limitations.

(1) The purpose of this chapter is to provide methods for estimating waves at one site given information at another. The assumption made is that *the wave information used as input to the analysis is characteristic of the waves that would propagate to the site*. In each case, the engineer should assure that there is no limitation of fetch, sheltering of waves, or oddness of bathymetry that would make selection of the input site inappropriate.

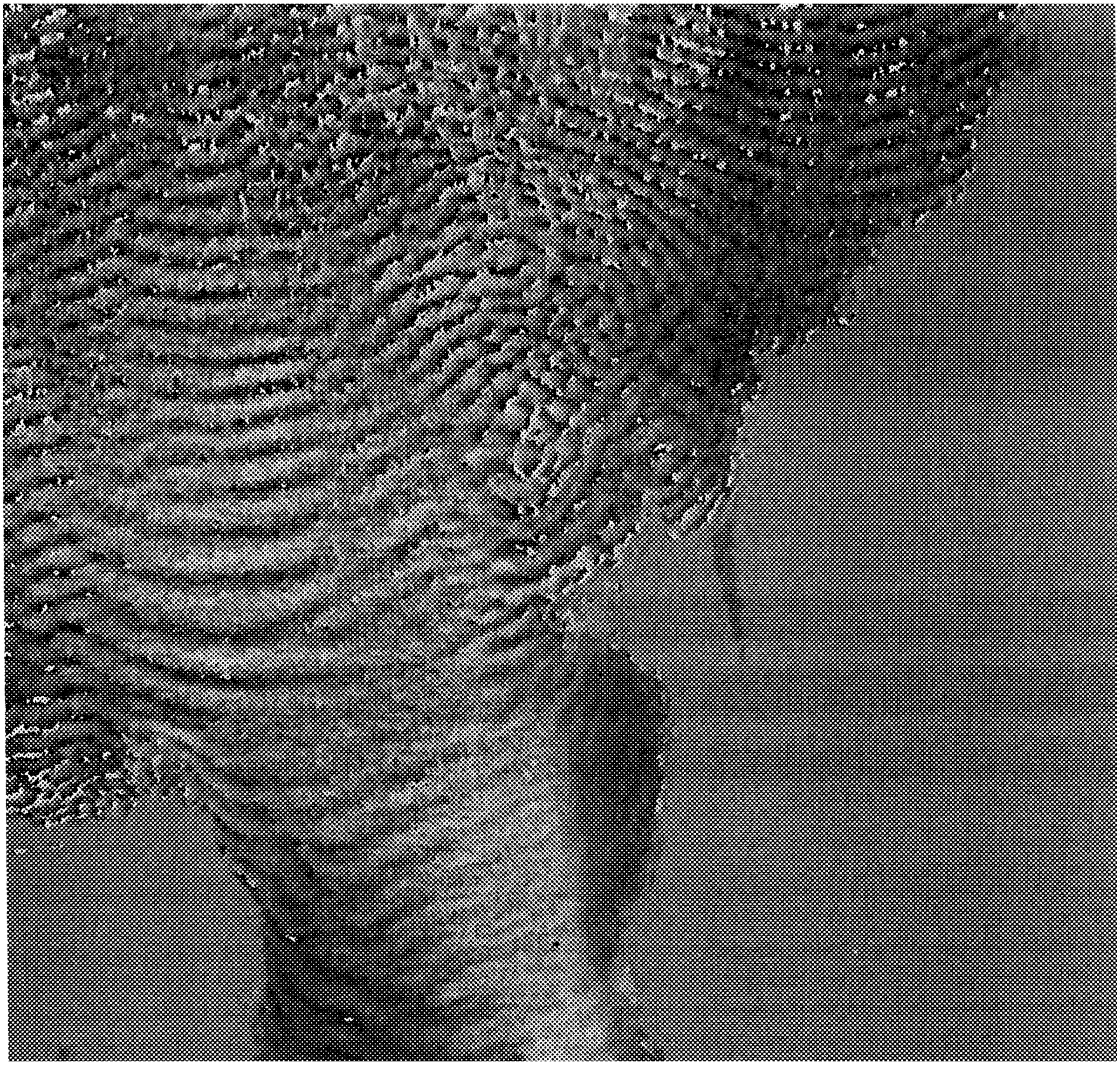

Figure II-3-1. Waves propagating through shallow water influenced by the underlying bathymetry and currents

(2) For most of the open U.S. coastline, Wave Information Study data or data from gauges provide adequate spacing of sites along the coast to give estimates of the wave climate that can be used as input to nearshore transformation studies. In other places or for simulation of a specific event, a special hindcast of the deepwater wave climate may be required to provide input for a transformation analysis.

c. Importance of water level. Near the coast, variable water depths can produce major variations in wave conditions over short distances. The important physical parameter is the depth of the water on which the surface waves are traveling. In nature, water depth is not a constant: it varies with tide stage, hurricane or extratropical storm surge, or for a variety of other reasons (Part II-5). These variations in water level influence wave breaking. *Hence, any study of wave transformation must account for expected water levels for the site and the situation of interest.*

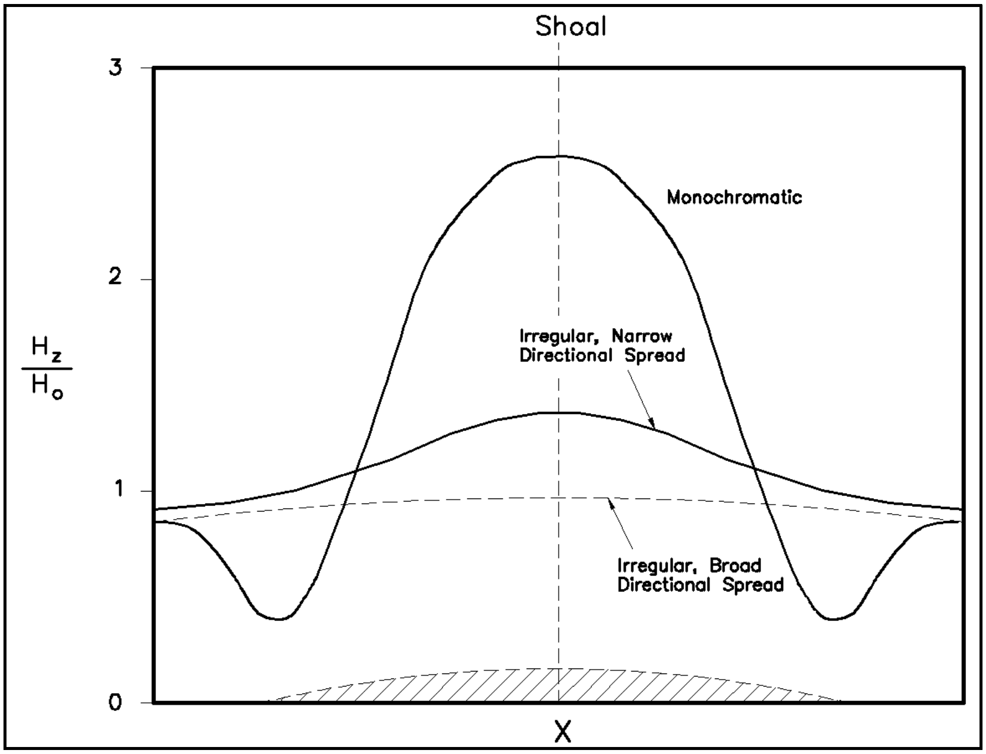

Figure II-3-2. Amplification of wave height behind a shoal for waves with different spreads of energy in frequency and direction

d. Role of gauging. The procedures described here are needed because long-term site-specific data often do not exist. If time and funding are available, a short-term gauging program should be considered. A gauging program can help in two ways:

(1) It may provide a simple statistically based transformation procedure.

(2) It can be used to validate/calibrate a numerical model as a transformation procedure for the project.

Even a few months of gauge data can be a significant complement to any wave-transformation analysis. Short-term gauging is generally not useful in providing, by itself, a design-wave height.

e. Physical modeling. This chapter emphasizes calculation procedures for estimating nearshore waves. However, some sites are so complicated that a physical model of the site may be required to determine the wave conditions. Physical modeling is a well-established procedure for analysis of wave propagation and breaking effects and is particularly useful in analysis of the effects of structures on the wave field. Physical modeling is not useful for evaluating bottom friction or percolation effects or inclusion of wind inputs. Because of scaling limitations and costs, physical models are generally used for small areas (a few square kilometers or less). If strong currents transverse to a wave field are present, such as at a tidal inlet, a physical model may be the only dependable method for estimating the wave field.

II-3-2. Principles of Wave Transformation

a. Introduction.

(1) In this section, the scientific principles governing the transformation of waves from deep water to shallow will be presented in sufficient detail to highlight critical assumptions and simplifications. Unfortunately, the problem is so complex that detailed computations require use of complicated numerical models whose background and implementation are beyond the scope of the Coastal Engineering Manual. This chapter provides the principles of wave transformation, a simplified approach, and an introduction to three numerical models used by the Corps of Engineers.

(2) Processes that can affect a wave as it propagates from deep into shallow water include:

(a) Refraction.

(b) Shoaling.

(c) Diffraction.

(d) Dissipation due to friction.

(e) Dissipation due to percolation.

(f) Breaking.

(g) Additional growth due to the wind.

(h) Wave-current interaction.

(i) Wave-wave interactions

The first three effects are *propagation* effects because they result from convergence or divergence of waves caused by the shape of the bottom topography, which influences the direction of wave travel and causes wave energy to be concentrated or spread out. Diffraction also occurs due to structures that interrupt wave propagation. The second three effects are *sink* mechanisms because they remove energy from the wave field through dissipation. The wind is a *source* mechanism because it represents the addition of wave energy if wind is present. The presence of a large-scale current field can affect wave propagation and dissipation. Wave-wave interactions result from nonlinear coupling of wave components and result in transfer of energy from some waves to others. The procedures presented will stop just seaward of the surf zone, which is treated in Part II-4, "Surf Zone Hydrodynamics."

b. Wave transformation equation.

(1) The general problem of wave transformation will be introduced in terms of the concept of directional wave spectra discussed in Part II-1 and II-2. Adopting the notation of Part II-2, consider a directional spectrum $E(x,y,t,f,\theta)$ where f,θ represents a particular frequency-direction component, x,y represents a location in geographic space, and t represents time. The waves are propagating over a region with varying water depths with no current. Water level will not be time-dependent in the following analyses. Structures are not considered. The general equation used to estimate wave transformation is the radiative transfer equation introduced in Part II-2.

$$\underset{A}{\frac{\partial E(x,y,t,f,\theta)}{\partial t}} + \underset{B}{\nabla \cdot \left[C_g(x,y,f)\ E(x,y,t,f,\theta) \right]} = \underset{C}{S_w} + \underset{D}{S_n} + \underset{E}{S_D} + \underset{F}{S_F} + \underset{G}{S_P} \qquad \text{(II-3-1)}$$

(2) Although multidimensional, this equation is fundamentally simple. Term *A* represents the temporal rate of change of the spectrum, term *B* represents the propagation of wave energy, term *C* represents inputs from the wind, term *D* represents the redistribution of wave energy between different wave components that arise from nonlinearities of the waves, term *E* represents dissipation due to breaking, term *F* represents losses due to bottom friction, and term *G* represents losses due to percolation. Many different algebraic forms have been suggested for the various S_i; three references that provide examples are WAMDI (1988), Sobey and Young (1986), and Young (1988). Since they are complicated and cannot be used in manual computations, their algebraic form is not provided here. More detailed discussion of spectral wave mechanics may be found in Leblond and Mysak (1978), Hasselmann (1962, 1963a, 1963b), Hasselmann et al. (1973), Barnett (1968), Phillips (1977), Resio (1981), WAMDI (1988), and in Parts II-1 and II-2.

(3) Surface wave motions produce a velocity field that extends to some depth in the water column. This depth for a deepwater wave is $L/2$ where L is the deepwater wave length. If the water depth is less than $L/2$, the motion extends to the bottom. In cases where the wave motion interacts with the bottom, several physical changes occur as shown in Part II-1: the celerity C and group velocity C_g are changed, as is the wavelength. If the waves are propagating in a region in which the depths are variable (and sufficiently shallow so that the wave interacts with the bottom), the changes in wave speed change the direction of wave travel and change the amplitude of the wave (*refraction and shoaling*). If the patterns of wave propagation lead to strong focusing of waves, wave energy may be radiated away from the convergence by diffraction (Penny and Price 1944; Berkhoff 1972). The interaction of the wave with the bottom produces a boundary layer, which will result in the loss of wave energy to the bed due to bottom friction (Term *F*) resulting from bottom materials and bed forms (Bagnold 1946). If the bed is reasonably porous, the pressure field associated with the passing wave can induce flow into and out of the bed (Bretschneider and Reid 1953), resulting in energy losses due to percolation (Term *G*). If the bed is muddy or visco-elastic other losses may occur (Forristall and Reece 1985). Typically, only one of the bottom loss mechanisms is dominant at one locality although in a large, complicated area a variety of bottom types may exist with differing mechanisms important at different sites along the path of wave travel. However, the bottom-loss terms are often not applied because inadequate information is available on bottom-material composition to allow their proper use.

(4) Wind input, interwave transfers, and breaking follow the principles outlined in Part II-2, though modified due to depth effects. Of the three, wave breaking is most affected by depth. If shoals exist, depth-induced breaking may be significant even though it is outside of the surf zone. Surf zone wave breaking is treated in Part II-4. The effect of sporadic breaking of large waves on shoals or other depth-related features outside the surf zone is not negligible in high sea states. Even in deep water, waves break through whitecapping or oversteepening due to superposition of large waves. The interaction of waves and an underlying current can result in refraction of the waves and wave breaking (Jonsson 1978; Peregrine 1976).

c. Types of wave transformation.

(1) Three classic cases of wave transformation describe most situations found in coastal engineering:

(a) A large storm generates deepwater waves that propagate across shallower water while the waves continue to grow due to wind.

(b) A large storm generates winds in an area remote from the site of interest and as waves cross shallower water with negligible wind, they propagate to the site as swell.

(c) Wind blows over an area of shallow water generating waves that grow so large as to interact with the bottom (no propagation of waves from deeper water into the site).

(2) All cases are important, but the first and third are relatively complex and require a numerical model for reasonable treatment. The second case, swell propagating across a shallow region, is a classic building block that has served as a basis for many coastal engineering studies. Often the swell is approximated by a monochromatic wave, and simple refraction and shoaling methods are used to make nearshore-wave estimates. Since the process of refraction and shoaling is important in coastal engineering, the next section is devoted to deriving some simple approaches to illustrate the need for more complex approaches.

(3) Often it is necessary for engineers to make a steady-state assumption: i.e., wave properties along the outer boundary of the region of interest and other external forcing are assumed not to vary with time. This is appropriate if the rate of variation of the wave field in time is very slow compared to the time required for the waves to pass from the outer boundary to the shore. If this is not the case, then a time-dependent model is required. Cases (a) and (c) would more typically require a time-dependent model. Time-dependent models are not discussed here due to their complexity. Examples are described by Resio (1981), Jensen et al. (1987), WAMDI (1988), Young (1988), SWAMP Group (1985), SWIM Group (1985), and Demirbilek and Webster (1992a,b).

II-3-3. Refraction and Shoaling

In order to understand wave refraction and shoaling, consider the case of a steady-state, monochromatic (and thereby long-crested) wave propagating across a region in which there is a straight shoreline with all depth contours evenly spaced and parallel to the shoreline (Figure II-3-3). In addition, no current is present. If a wave crest initially has some angle of approach to the shore other than 0 deg, part of the wave (point A) will be in shallower water than another part (point B). Because the depth at A, h_A, is less than the depth at B, h_B, the speed of the wave at A will be slower than that at B because

$$C_A = \frac{g}{\omega} \tanh k h_A < \frac{g}{\omega} \tanh k h_B = C_B \qquad \text{(II-3-2)}$$

The speed differential along the wave crest causes the crest to turn more parallel to shore. The propagation problem becomes one of plotting the direction of wave approach and calculating its height as the wave propagates from deep to shallow water. For the case of monochromatic waves, wave period remains constant (Part II-1). In the case of an irregular wave train, the transformation process may affect waves at each frequency differently; consequently, the peak period of the wave field may shift.

a. Wave rays.

(1) The wave-propagation problem can often be readily visualized by construction of wave rays. If a point on a wave crest is selected and a wave crest orthogonal is drawn, the path traced out by the orthogonal as the wave crest propagates onshore is called a ray. Hence, a group of wave rays map the path of travel of the wave crest. For simple bathymetry, a group of rays can be constructed by hand to show the wave transformation, although it is a tedious procedure. Graphical computer programs also exist to automate this process (Harrison and Wilson 1964, Dobson 1967, Noda et al. 1974), but to a large degree such approaches

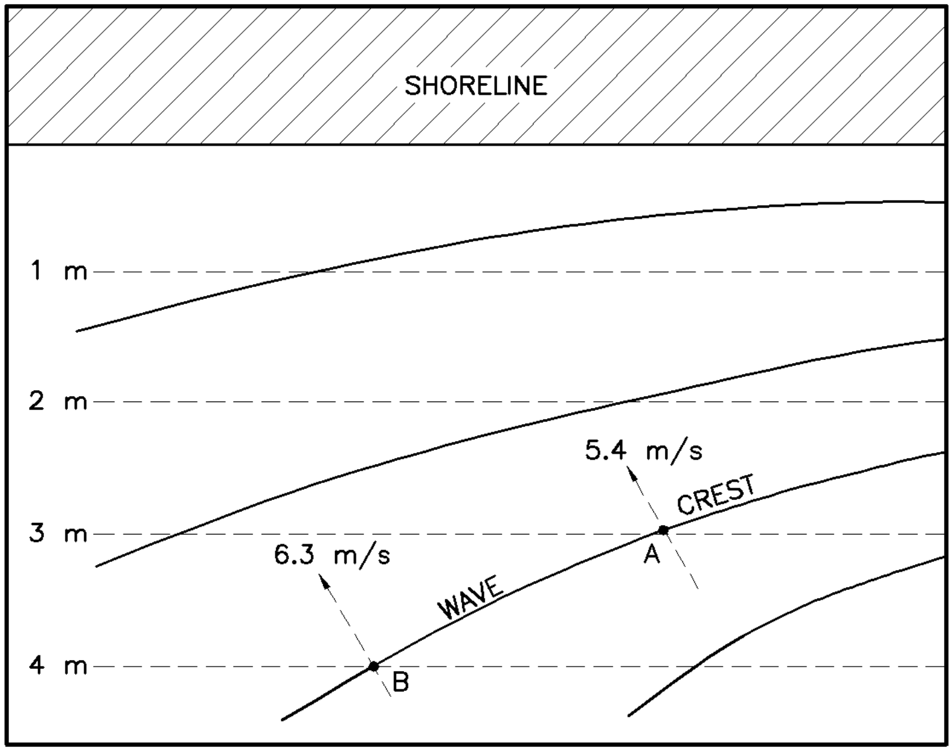

Figure II-3-3. Straight shore with all depth contours evenly spaced and parallel to the shoreline

have been superseded by the numerical methods discussed in Part II, Section 3-5. Refraction and shoaling analyses typically try to specify the wave height and direction along a ray.

(2) Figure II-3-4 provides idealized plots of wave rays for several typical types of bathymetry. Simple parallel contours tend to reduce the energy of waves inshore if they approach at an angle. Shoals tend to focus rays onto the shoals and spread energy out to either side. Canyons tend to focus energy to either side and reduce energy over the head of the canyon. The amount of reduction or amplification will depend not only on bathymetry, but on the initial angle of approach and period of the waves. For natural sea states that have energy spread over a range of frequencies and directions, reduction and amplification are also dependent upon the directional spread of energy (Vincent and Briggs 1989).

(3) Refraction and shoaling have been derived and treated widely. The following presentation follows that of Dean and Dalrymple (1991) very closely. Other explanations are provided in Ippen (1966), the *Shore Protection Manual* (1984), and Herbich (1990).

b. Straight and parallel contours.

(1) First, the equation for specifying how wave angle changes along the ray is developed, followed by the equation for wave height. The derivation is only for parallel and straight contours with no currents present. The x-component of the coordinate system will be taken to be orthogonal to the shoreline; the y-coordinate is taken to be shore-parallel. The straight and shore-parallel contours assumption will imply that any derivative in the y-direction is zero because *dh/dy* is zero.

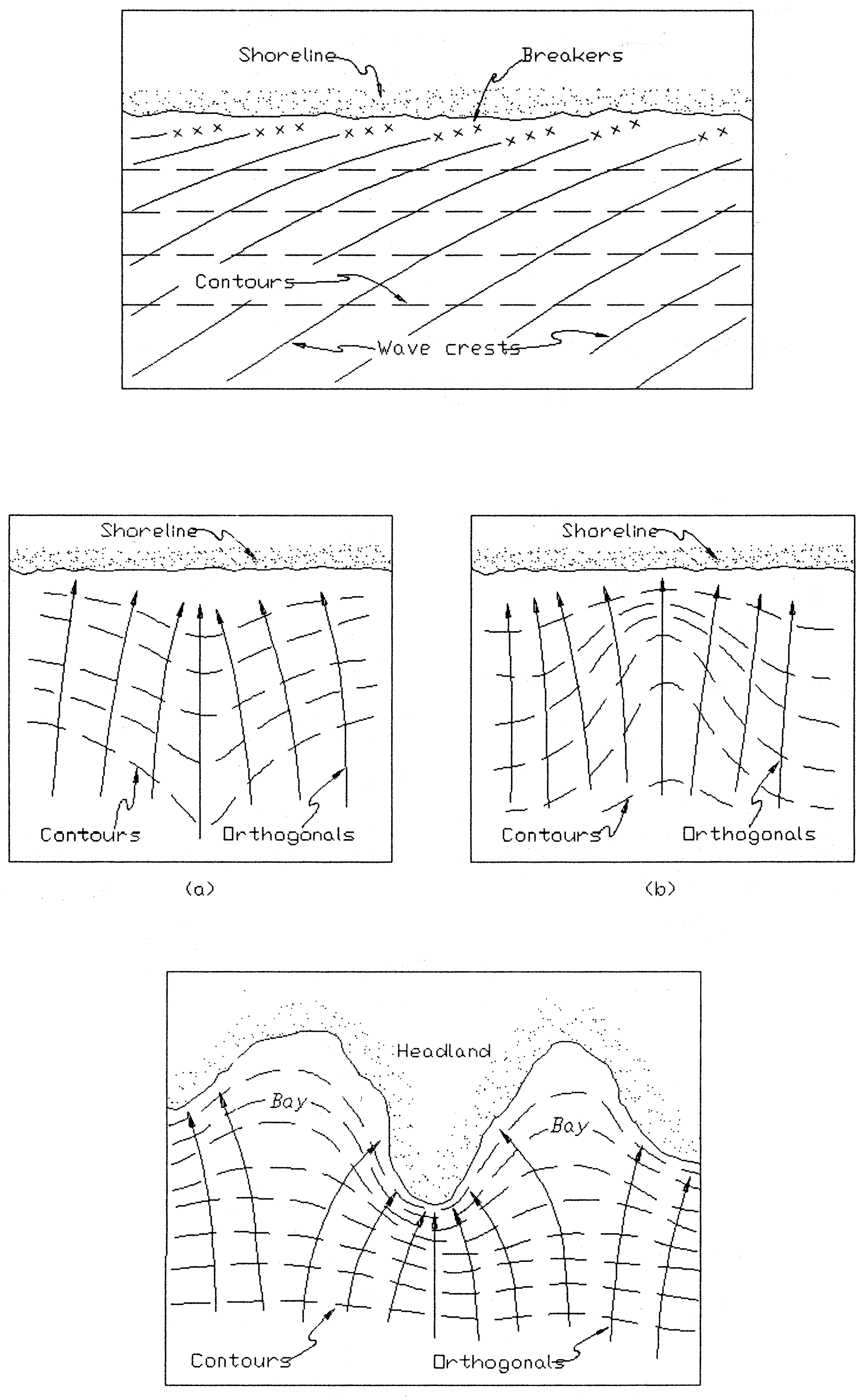

Figure II-3-4. Idealized plots of wave rays

(2) For a monochromatic wave, the wave phase function

$$\Omega\ (x,y,t) = (k\ \cos\theta + k\ \sin\theta - \omega t) \tag{II-3-3}$$

can be used to define the wave number vector $\vec{k}$ by

$$\vec{k} = \nabla\ \Omega \tag{II-3-4}$$

(3) Since $\vec{k}$ is a vector, one can take the curl of $\vec{k}$

$$\nabla \times \vec{k} = 0 \tag{II-3-5}$$

which is zero because $\vec{k}$ by definition is the gradient of a scaler and the curl of a gradient is zero.

(4) Substituting the components of $\vec{k}$, Equation II-3-5 yields

$$\frac{\partial(k\ \sin\theta)}{\partial x} - \frac{\partial(k\ \cos\theta)}{\partial y} = 0 \tag{II-3-6}$$

(5) Since the problem is defined to have straight and parallel contours, derivatives in the y direction are zero and using the dispersion relation linking k and C (and noting that $k = 2\pi/CT$ and wave period is constant) Equation II-3-6 simplifies to

$$\frac{d}{dx}\left(\frac{\sin\theta}{C}\right) = 0 \tag{II-3-7}$$

or

$$\frac{\sin\ \theta}{C} = constant \tag{II-3-8}$$

(6) Let C_0 be the deepwater celerity of the wave. In deep water, sin $(\theta_0)/c_0$ is known if the angle of the wave is known, so Equation II-3-8 yields

$$\frac{\sin\ \theta}{C} = \frac{\sin\ \theta_0}{C_0} \tag{II-3-9}$$

along a ray. This identity is the equivalent of Snell's law in optics. The equation can be readily solved by starting with a point on the wave crest in deep water and incrementally estimating the change in C because of changes in depth. The direction θ of wave travel is then estimated plotting the path traced by the ray. The size of increment is selected to provide a smooth estimate of the ray.

(7) The wave-height variation along the ray can be estimated by considering two rays closely spaced together (Figure II-3-5). In deep water, the energy flux (EC_n), which is also EC_g, across the wave crest distance b_0 can be estimated by $(ECn)_0 b_0$. Considering a location a short distance along the ray, the energy flux is $(ECn)_1 b_1$. Since the rays are orthogonal to the wave crest, there should be no transfer of energy across the rays and conservation principles give

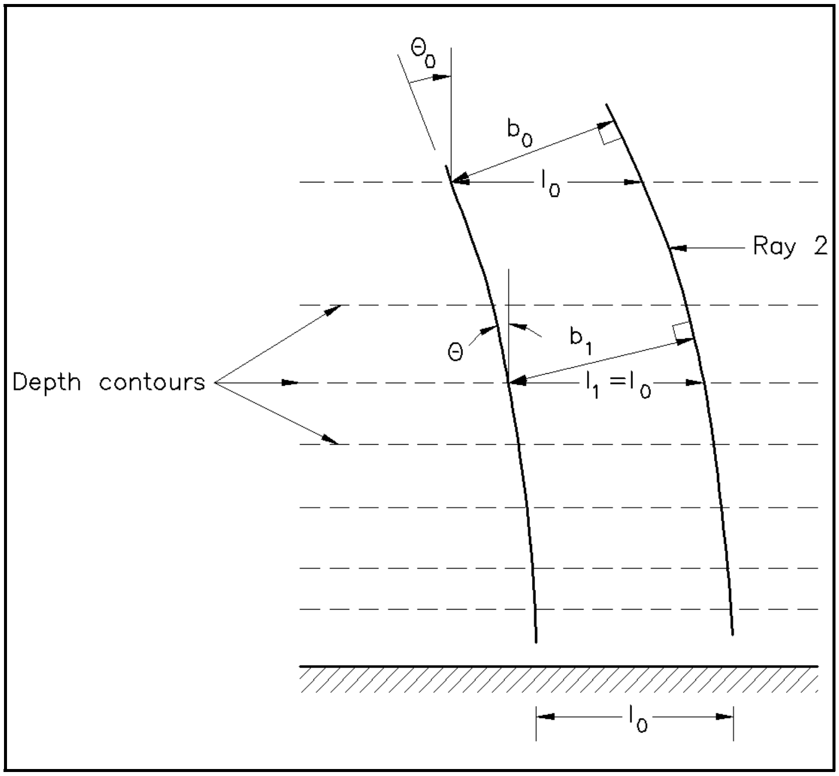

Figure II-3-5. Wave-height variation along a wave ray

$$(ECn)_0 b_0 = (ECn)_1 b_1 \tag{II-3-10}$$

(8) From Part II-1, the height and energy of a monochromatic wave are given by

$$E = \frac{1}{8} \rho g H^2 \tag{II-3-11}$$

and the wave height at location 1 is thus related to the wave height in deep water by

$$H_1 = H_0 \sqrt{\frac{C_{g_0}}{C_{g_1}}} \sqrt{\frac{b_0}{b_1}} \tag{II-3-12}$$

(9) This equation is usually written as

$$H_1 = H_0 \, K_s \, K_r \tag{II-3-13}$$

where K_s is called the shoaling coefficient and K_r is the refraction coefficient. From the case of simple, straight, and parallel contours, the value at b_1 can be found from b_0

$$K_r = \left(\frac{b_o}{b_1} \right)^{\frac{1}{2}} = \left(\frac{\cos\ \theta_0}{\cos\ \theta_1} \right)^{\frac{1}{2}} = \left(\frac{1 - \sin^2\ \theta_0}{1 - \sin^2\ \theta_1} \right)^{\frac{1}{4}} \tag{II-3-14}$$

by noting that ray 2 is essentially ray 1 shifted downcoast. For straight and parallel contours, Figure II-3-6 is a solution nomogram. This is automated in the ACES program (Leenknecht, Szuwalski, and Sherlock 1992) and the program NMLONG (Kraus 1991). Figure II-3-6 provides the local wave angle K_R and $K_R K_S$ in terms of initial deepwater wave angle and d/gT^2. Although the bathymetry of most coasts is more complicated than this, these procedures provide a quick way of estimating approximate wave approach angles.

c. Realistic bathymetry.

(1) The previous discussion was for the case of straight and parallel contours. If the topography has variations in the y direction, then the full equation must be used. Dean and Dalrymple (1991) show the derivation in detail for ray theory in this case. Basically, the (x,y) coordinate system is transformed to (s,n) coordinates where s is a coordinate along a ray and n is a coordinate orthogonal to it. Algebraically, the equation for wave angle can be derived in the ray-based coordinate system

$$\frac{\partial \theta}{\partial s} = \frac{1}{k}\frac{\partial k}{\partial n} = -\frac{1}{C}\frac{\partial C}{\partial n} \tag{II-3-15}$$

and the ray path defined by

$$\frac{ds}{dt} = C \tag{II-3-16}$$

$$\frac{dx}{dt} = C \cos\theta \tag{II-3-17}$$

$$\frac{dy}{dt} = C \sin\theta \tag{II-3-18}$$

(2) Equation II-3-15 represents the discussion at the beginning of this section; the rate at which the wave turns depends upon the local gradient in wave speed along the wave crest. Munk and Arthur's computation for the refraction coefficient is more complicated: defining

$$K_r = \left(\frac{1}{\beta}\right)^{\frac{1}{2}} \tag{II-3-19}$$

where $\beta = b/b_0$ then

$$\frac{d^2\beta}{ds^2} + p\,\frac{d\beta}{ds} + q\beta = 0 \tag{II-3-20}$$

with

$$p(s) = -\frac{\cos\theta}{C}\frac{\partial C}{\partial x} - \frac{\sin\theta}{C}\frac{\partial C}{\partial y} \tag{II-3-21}$$

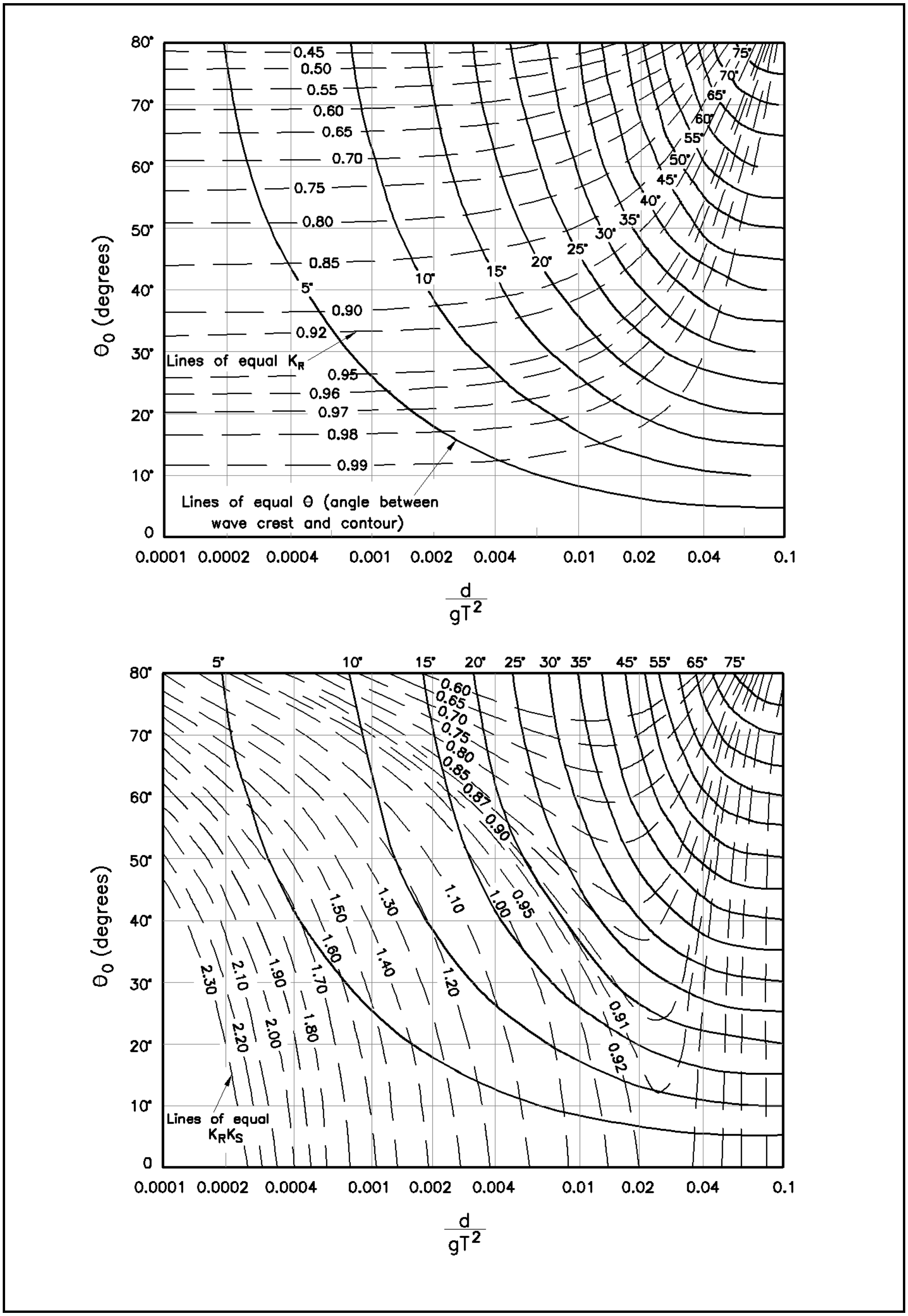

Figure II-3-6. Solution nomogram

EXAMPLE PROBLEM II-3-1

FIND:

Wave height H and angle θ at water depths of 200, 100, 90, 80, 70, 60, 50, 40, 30, 20, 10, 16, 14, 12, 10, 8, 6, and 4 m for deepwater wave angles of 0°, 15°, and 45°.

GIVEN:

A wave 1 m high and 15-sec period in 500 m of water, with a plane, sloping beach.

SOLUTION:

Routine solutions for a plane beach can be obtained using the ACES wave transformation code, by direct calculation, or graphically using Figure II-3-6.

Table II-3-1 provides the results obtained by directly using the ACES code. On a personal computer with a 486-level microprocessor, the results may be obtained in seconds.

For a wave with a depth of 10 m and an initial wave angle of 45 deg, wave height and angle are calculated as follows:

Since the deepwater wave length of a 15-sec wave is

$$L_0 = 1.56\ T^2 = 1.56\ (15)^2 = 351\ m$$

and since 500 m is greater than $L_0/2$, the given initial wave is a deepwater condition. The wave length of the wave in 10 m must be estimated from

$$L = \frac{g\ T^2}{2\ \pi} \tanh \left(\frac{2\ \pi\ d}{L} \right)$$

and is 144 m (see Problem II-1-1).

The shoaling coefficient K_s can be estimated from

$$K_s = \left(\frac{C_{g0}}{C_{gl}} \right)^{\frac{1}{2}}$$

In deep water C_{g0} for a 15-sec wave is

$$\frac{1}{2}\ C_0 = \frac{1}{2}\ (1.56\ T) = \frac{23.4}{2} = 11.7\ \mathrm{m/s}$$

The group velocity is given by

$$C_g = n\,C = \frac{1}{2} \left(1 + \frac{4\pi d/L}{\sinh\ (4\pi d/L)} \right) \frac{g\,T}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$$

Substitution of d = 10 m, L = 144 m, T = 15 sec, and g = 9.8 m/sec^2 yields 9.05 m/s.

$$K_s = \left(\frac{11.70}{9.05} \right)^{\frac{1}{2}} = 1.14$$

Solution for K_r involves

$$K_r = \left(\frac{1 - \sin^2\theta_0}{1 - \sin^2\theta_1} \right)^{\frac{1}{4}}$$

(Continued)

Example Problem II-3-1 (Concluded)

In deep water, θ is 45 deg. From Equation II-3-9,

$$\sin\ \theta\ =\ \frac{C_1\ \sin\ \theta_0}{C_0}$$

In deep water C_0 = 1.56T = 23.4 m/s. In 10 m of water, $C_1 = L_1/T$ = 144 m/15$_s$ = 9.60 m/s.

$$\sin\ \theta\ =\ \frac{9.6\ \sin\ (45^0)}{23.4}\ =\ \frac{9.6\ (7.07)}{23.4}\ =\ 0.29$$

$$K_r\ =\ \left(\frac{1\ -\ \sin^2\theta_0}{1\ -\ \sin^2\theta}\right)^{\frac{1}{4}}\ =\ \left(\frac{1\ -\ (0.707)^2}{1\ -\ (0.29)^2}\right)^{\frac{1}{4}}\ =\ \left(\frac{0.50}{0.91}\right)^{\frac{1}{4}}\ =\ 0.86$$

Therefore: $H_i = H_0\, K_s\, K_r$ = 1(1.14) (0.86) = 0.98 m.

The angle of approach is arc sin (sinθ) = 16.8^0. Thus, the l-m, 15-sec wave has changed 2 percent in height by 28.2 deg in angle of approach.

The largest differences caused by refraction and shoaling will be seen at the shallowest depths. From Table II-3-1 at the 4-m depth, the wave height for a 45-deg initial angle is 1.18 m compared to 1.39 m for a wave with initial angle of 0 deg. If the initial angle had been 70 deg, $K_r\, K_s$ would be about 0.8.

Table II-3-1
Example Problem II-3-1 Refraction and Shoaling Results

Depth	$\theta_o = 0°$		$\theta_o = 15°$		$\theta_o = 45°$	
	θ	H	θ	H	θ	H
500	0	1.00	15.0	1.00	45.0	1.00
400	0	1.00	15.0	1.00	45.0	1.00
300	0	1.00	15.0	1.00	45.0	1.00
200	0	1.00	15.0	1.00	45.0	1.00
100	0	0.94	14.3	0.94	42.4	0.92
90	0	0.93	14.0	0.93	41.2	0.91
80	0	0.93	13.7	0.92	30.4	0.89
70	0	0.92	13.2	0.91	38.9	0.88
60	0	0.91	12.7	0.91	37.0	0.86
50	0	0.91	12.0	0.91	34.5	0.85
40	0	0.92	11.1	0.92	31.8	0.84
30	0	0.95	9.9	0.94	28.1	0.85
20	0	1.00	8.4	0.99	23.4	0.88
18	0	1.02	8.0	1.01	22.3	0.89
16	0	1.04	7.8	1.03	21.1	0.91
14	0	1.07	7.1	1.05	19.8	0.92
12	0	1.10	6.6	1.08	18.4	0.95
10	0	1.14	6.1	1.12	16.8	0.98
8	0	1.19	5.5	1.17	15.1	1.02
6	0	1.27	4.8	1.25	13.15	1.08
4	0	1.39	3.9	1.37	10.8	1.18

and

$$q(s) = \frac{\sin^2\theta}{C}\frac{\partial^2 C}{\partial x^2} - 2\frac{\sin\theta\,\cos\theta}{C}\frac{\partial^2 C}{\partial x\,\partial y} + \cos^2\theta\frac{\partial^2 C}{\partial y^2} \qquad \text{(II-3-22)}$$

(3) These equations are solved for a set of rays for each wave component of interest (typically combinations of periods and directions). Since this analysis is linear, often a unit wave height is applied for the offshore wave height, which yields a series of refraction and shoaling coefficients at sites of interest. Then the wave transformation for any non-unit initial wave height is obtained by multiplication. This is permissible as long as wave breaking does not occur along a wave ray.

d. Problems in ray approach.

(1) Estimating wave propagation patterns with wave rays is intuitively and visually satisfying, and often very useful. The engineer obtains a good picture of how a wave propagates to a site. However, the procedure has several drawbacks when applied to even mildly irregular bathymetry. One problem is ray convergence/crossing; another is bathymetry inadequacy on ray paths.

(2) An example calculation from Noda et al. (1974) illustrates the basic problem. Bathymetry is highly regular, but has undulatory contours (Figure II-3-7). From the ray pattern, convergence and divergence of adjacent rays are apparent as the waves sweep over the undulations in bathymetry. However, in shallow water near the shore, the rays are sufficiently perturbed by the bathymetry that several converge, with the ray spacing going to zero (in some ray programs the rays actually are computed to cross). Remembering the conservation of wave energy argument used to define the refraction coefficient, the flux across an orthogonal between the rays remains constant. As the spacing between rays approaches zero, the energy flux becomes infinite. Practically, if strong wave convergence occurs, breaking either due to depth constraints (Part II-4) or steepness constraints (Part II-1) naturally limits the wave height. However, situations which generate strong gradients or discontinuities in wave height along a wave crest give rise to ***diffraction*** effects, which can reduce the wave height and keep it below the breaking value.

(3) The second problem with ray theory is the sensitivity of the wave ray calculations. In most locations, the bathymetry is not well-known. Discretizations of the bathymetry can produce sharper local gradients in the computational depth field than may exist locally or, conversely, may reduce local gradients. Most wave ray calculation schemes calculate each wave ray uncoupled from all others. Ray paths are very sensitive to gradients in bathymetry. The smoothing algorithms that are used to numerically compute the required derivatives can alter the ray field significantly. Since the ray calculations are uncoupled, adjacent rays may take radically different paths due to how the bathymetry was discretized or smoothed. Also, if the ray calculations were started at slightly different spatial locations, the resulting patterns may be significantly different for the same reason. *In the cases where ray patterns are unstable with respect to perturbations of initial positions or where adjacent rays show unusual divergences and crossings, the coastal engineer must carefully assess whether the propagation is indeed that unusual (in which case ray theory results may not be accurate) or decide that more careful analysis of the bathymetry and smoothing is needed.*

(4) Wave propagation discussion has centered on the concept of waves traveling from deep water to shallow. At some locations, the bathymetry is such that waves propagating from offshore towards a beach may initially propagate from deeper to shallow water, then propagate across a zone where the water becomes deeper again. In the region where the wave is propagating at an angle to the progressively shallower depths, the process of refraction previously described occurs: the waves turn more shore-normal. Once the depth gradient reverses, the wave turns in the opposite direction (because of the reversed depth gradient, the part

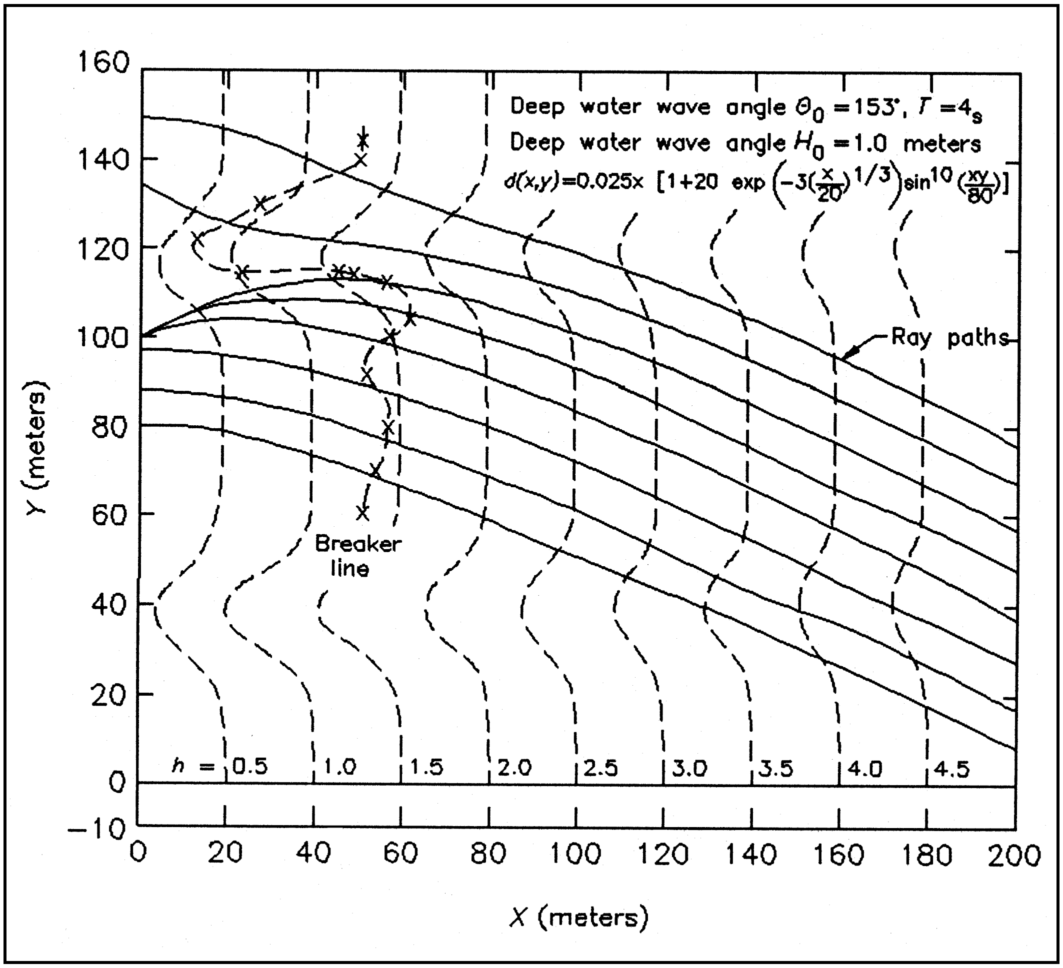

Figure II-3-7. Highly regular bathymetry but undulatory contours

of the wave crest nearer shore is in deeper water than the part of the wave offshore and is hence moving faster). If the combination of wave angle of approach to the bathymetry and wave period is correct, the wave will turn back and go offshore, giving the appearance of a reflected wave. Calculation of wave heights where waves crook or bend backwards is not treated here.

e Wave diffraction. Wave diffraction is a process of wave propagation that can be as important as refraction and shoaling. The classical introduction to diffraction treats a wave propagating past the tip of a breakwater. Since diffraction theory is most often applied to the interaction of waves with harbor structures, derivation of wave diffraction is deferred until Part II-7 (Harbor Hydrodynamics). Any process that produces an abrupt or very large gradient in wave height along a wave crest also produces diffracted waves that tend to move energy away from higher waves to the area of lower waves. So initial wave energy is reduced as diffracted waves are produced. However, if the rate of convergence is too great, the wave may still break.

f. Reflection. Waves that propagate into a solid object such as a breakwater, a seawall, a cliff, or a sloping beach may reflect. In the case of a vertical, hard structure, the fraction of wave energy reflected can be large. For permeable structures or gentle slopes, the reflection will be much less. For nearshore wave propagation problems, reflections are usually ignored because the reflected wave may often be less than 10 percent of the incident wave.

g. Refraction and shoaling of wave spectra. The previous discussion of refraction and shoaling was for a single wave component. However, wave propagation was introduced in Equation II-3-1 through the concept of spectral components. In principle, refraction and shoaling of a wave field in terms of its spectral components simply requires computing the refraction and shoaling coefficient for each frequency-direction (f,θ) component and computing the transformed sum:

$$E(f,\theta) = \sum K_s^2(f,\theta) \quad K_r^2(f,\theta) \quad E_o(f,\theta) \quad \Delta f \, \Delta\theta \tag{II-3-23}$$

where $E_o(f,\theta)$ is the offshore directional spectrum. This is possible as long as no breaking or other loss or gain occurs along the propagation path of the individual waves. If it does occur, most advanced spectral models compute the wave transformation locally. In this approach, the area of interest is covered by a discrete series of computation points and the ray path for each (f,θ) component in the spectrum is computed for each grid point only by tracing the ray back to the grid cell boundary defined by adjacent grid points. This approximation, called backward ray tracing, is adequate as long as the wave energy and bathymetry vary smoothly and gently over the domain.

h. Alternate formulations.

(1) Mild slope equation. The refraction and shoaling analyses presented above were based on linear wave theory and a ray approach equivalent to geometrical optics. This works well for simple cases, but once the bathymetry becomes even moderately undulatory, the ray approach runs into difficulty. Berkhoff (1972) formulated a more advanced approach for wave propagation that includes refraction, shoaling, and diffraction simultaneously and can incorporate structures. Berkhoff developed what is termed the mild-slope equation given by

$$\nabla \, (CC_g \Phi) + \omega^2 \left(\frac{C_g}{C} \right) \Phi = 0 \tag{II-3-24}$$

with

$$\varphi(x,y,z) = \Phi \, \frac{\cosh \, k(h+z)}{\cosh \, kh} \tag{II-3-25}$$

where

$$\nabla = \left(\frac{\partial}{\partial x_i} \, , \, \frac{\partial}{\partial y_j} \right) \tag{II-3-26}$$

which provides a solution φ for amplitude and phase of the waves in the horizontal plane. To obtain the equation, Berkhoff assumed that the bottom slope was mild (no abrupt steps, shoals, or trenches). Often slopes of interest violate this assumption, but the models based on the mild-slope equation perform better than the ray approach. Many approaches have been taken to computationally solve this equation. Berkhoff's approach solves the velocity potential of the wave in the horizontal, which can require 5-10 computational grid points per wave length. This is impractical for many cases. Another approach, developed by Radder (1979), is to use a parabolic approximation, which is far more computationally efficient (but subsequently adds more limitations).

(2) Boussinesq equations. Another approach for wave propagation problems close to the coast and in harbors is the use of vertically integrated shallow-water equations in which a Boussinesq (Part II-1) approximation has been made. The numerical models (e.g., Abbott, Peterson, and Skovgaard 1978) resulting from this approach require 10-20 grid points per wave length but have the advantage of being time-dependent

so that the pattern of wave propagation can be directly visualized. Wave crests evolve during the shoaling process to have nonsinusoidal shapes characteristic of shallow-water waves. Currents may be applied directly. Wave breaking, however, is simulated empirically.

II-3-4. Transformation of Irregular Waves

a. The preceding discussion emphasized the refraction and shoaling of monochromatic waves. When this process is applied to an initial significant wave height and period, it is called a *significant wave analysis.* For many conditions where propagation is the dominant factor (as opposed to additional wave growth or bottom dissipation) the significant wave analysis provides a reasonable and generally conservative approximation. The significant wave analysis may be inadequate when wave conditions have spectra characterized by wide directional spreads or broad (frequency) spectral widths or multiple spectral peaks. Cases where the significant wave analysis is adequate primarily involve narrow band swell. This section outlines differences that may be expected between the application of significant wave analyses and application of an irregular wave approach.

b. Rrefraction and shoaling for monochromatic waves may be applied to the individual frequency and direction components of the spectrum of an irregular wave system. Two factors become important: directional spreading and spectral wave mechanics. Directional spreading is important whenever it is present. Spectral wave dynamics are most important in high-energy, high-steepness wave cases, and negligible for low-energy, low-steepness cases.

c. Directional spreading is important for two reasons. First, laboratory tests (Berkhoff, Boij, and Radder 1982; Vincent and Briggs 1989) with unidirectional waves indicate that shoals concentrate wave energy immediately behind the shoal and reduce it on the flanks. The increase behind the shoal can be nearly 250 percent of the initial wave height; the reduction to either side can be about 50 percent. However, laboratory tests with wave spectra having significant directional spread (Vincent and Briggs 1989) show only a 110- to 140-percent increase behind the shoal and only a 10- to 15-percent reduction on the sides. Numerical models incorporating directional spread also replicate this (Panchang et al. 1990). In the case with directional spread, the shoal focusses each frequency-direction component at a different location behind the shoal rather than at one spot as in the unidirectional case. Consequently some of the high- and low-energy regions overlap and cancel each other out. Secondly, if the mean angle of wave approach is not directly onshore, one consequence of directional spreading is that some fraction of the wave energy is heading parallel to shore or offshore. In the case of a wave system with symmetric directional spread (i.e., 50 percent to the left and right of the mean direction), if the mean direction were parallel to a straight shoreline (and the measurement were made in deep water), half of the energy would be moving in directions that could not refract towards shore. So even for angles up to 30 deg offshore-parallel, significant amounts of energy are not propagating shoreward. In a significant wave analysis, all the energy would propagate shoreward. If the shoreline, fetch, or bathymetry is complicated, the fraction of energy that propagates towards shore is more difficult to define.

d. Spectral dynamics arise because waves of different lengths and steepness are propagating through and with other waves. According to Equation II-3-1, these waves can exchange energy between each other (nonlinear transfers) and superposition of waves can lead to dissipation due to breaking. Analysis of thousands of wave records (Bouws et al. 1987; Bouws, Gunther, and Vincent 1985; Miller and Vincent 1990) indicates that higher energy wind sea spectra achieve a characteristic shape that is different from that obtained simply by shoaling. As a result, the energy level for shoaling irregular wave tends to be less than that predicted from linear monochromatic shoaling of the wave components, especially near the surf zone. Smith and Vincent (1992) also indicate that the shoaling and breaking of irregular waves with two spectral peaks can substantially differ from the monochromatic (and even single peak spectral) case. Moreover, the wave spectrum after refraction and shoaling can have a substantially different peak period. Although a satisfactory explanation of these phenomena is not available, their impact is significant, with differences up to 30-40 percent from the significant wave approach.

e. Treatment of spectral wave mechanics in any detail requires use of a numerical model. However, in using a significant wave approach, it can generally be assumed that:

(1) It may overestimate wave focussing effects.

(2) Careful estimates of the fraction of wave energy heading shoreward should be made for oblique angles cases.

(3) Shoaling calculations may overestimate wave heights in high energy conditions.

f. Shifts in wave period may also occur. As a result, significant wave analysis tends to be conservative; this may be why it has been an acceptable approach for design. However, for cost-sensitive projects, a more complicated approach may be warranted.

g. The following precautions are suggested. In a significant wave analysis, if regions of highly focussed wave energy occur with corresponding lobes of low energy, the regions of low energy should be carefully considered. In the field, natural wave systems generally have significant directional spread, so calculated values in the low energy lobes may significantly underestimate wave heights. In cases where irregular waves are modeled spectrally, typically only the wave height H_s is estimated. In shallow water, larger waves do occur ($H_{1/10}$, etc.) and combinations of individual wave height, period, and bottom depth can result in individual waves or groups of waves significantly larger than H_s (see Part II-2).

II-3-5. Advanced Propagation Methods

a. Introduction.

(1) As indicated in Part II-3-2, as waves propagate, they may continue to grow due to the continued action of the wind or may lose energy due to breaking, bottom friction, or percolation. These effects cannot be realistically incorporated through manual calculations. The preceding discussion indicates that computations involving rays are tedious by hand and subject to many inaccuracies. Advances have been made in computing wave transformation; they were briefly indicated in the preceding sections. Many of these procedures may run efficiently on a personal computer or a work station and do not require a large mainframe or supercomputer. Hence they can be applied readily by most engineers (ACES 1992).

(2) This section describes three computer programs that are available and in use by the Corps of Engineers. Each program is briefly described and a reference indicates where the program can be obtained. Each program is complicated and requires some effort to use properly. A short description is provided here to indicate to the engineer the potentials of these codes. The three have been selected to provide a cross section of the types of technology available. Other computer programs can be obtained and may be as suitable for use as those described here.

(3) Examples of technology available to practicing engineers is provided. **The Corps of Engineers does not endorse the codes discussed or certify their accuracy.** Indeed, the suitability and accuracy of any of these codes depend upon the problem under study and the way in which the code is applied. **With the exception of very simple bathymetry, it is recommended that nearshore wave transformation studies use a numerical code capable of handling the complexities required.** However, the particular numerical approach selected depends upon the problem.

(4) The three models discussed below are all steady-state models. Time-dependent, shallow-water models are available (Jensen et al. 1987; Demirbilek and Webster 1992a, 1992b). They are not discussed here because they require extensive sets of meteorological data and cannot be easily applied. The basic characteristics of the three models discussed are as follows:

(a) (RCPWAVE) RCPWAVE is a steady-state, linear-wave model based on the mild-slope equation and includes wave breaking. It is applicable for open coast areas without structures. It is basically a monochromatic-wave approach.

(b) (REFDIF1) REFDIF is a steady-state model based on the parabolic approximation solution to the mild-slope equation. The model includes wave breaking, wave damping, and some nonlinear effects. Although primarily used as a monochromatic wave model, a spectral version is available. The model can simulate aspects of propagation associated with simple currents and can include structures.

(c) (STWAVE) STWAVE is a steady-state, linear wave model that computes the evolution of the directional spectrum over space (Equation II-3-1). The model includes breaking, bottom friction, percolation, and wind input and solves for the nonlinear transfers of energy within the wave spectrum. It has two modes for handling diffraction of wave energy and the computational domain may include simple structures. The models can handle aspects of propagation associated with simple currents.

(5) The three models are theoretically complicated and computationally demanding. All can be effectively used on a powerful PC-type computer or work station. Each model has considerable strengths and each can be an appropriate choice for wave transformation. However, none can be considered universally applicable and the results from all can be inaccurate if the assumptions made in model development are significantly violated. Users of any of the models must become thoroughly familiar with the model, its assumptions, and limitations.

b. RCPWAVE.

(1) Introduction.

(a) The RCPWAVE model (Ebersole 1985; Ebersole, Cialone, and Prater 1986) was developed in the early 1980's as an engineering tool for calculating the properties of waves as they propagate into shallow water and eventually break. The theoretical basis for the model (linear-wave theory) and the types of information generated by the model (wave height, period, and direction as a function of location) are consistent with current theories and equations used by the engineering community to calculate potential longshore sand-transport rates and shoreline and beach change. The model was designed to operate efficiently for coastal regions that may be tens of kilometers in length, and to overcome deficiencies of previously developed refraction models that could be applied on a regional scale. The wave ray refraction models of Harrison and Wilson (1964), Dobson (1967), Noda et al. (1974), and others "failed" in regions of strong wave convergence and divergence (i.e., highly irregular bathymetry), leaving users with no wave solutions and little guidance for interpreting results in these regions. Berkhoff (1972, 1976) derived an elliptic equation that approximately represented the complete transformation process for linear waves over arbitrary bathymetry, where the bathymetry was only constrained to have mild slopes. Numerical solution of this equation requires discretization of the spatial domain and subsequent computations with grid resolutions that are a fraction of the wave lengths being considered (typically one tenth or smaller). This requirement limits the utility of the approach for large regions of coastline.

(b) RCPWAVE is based on the mild-slope equation. An assumed form for the velocity potential associated with only the forward scattered wave field is used with the mild slope equation to develop two equations, one describing the conservation of wave energy (assuming a constant wave frequency) and the